Abstract

In the context of seismic observations in the nearfield of the source or when observing the motion of structures the response of pendulum seismometers to rotary motion is discussed occasionally. However, seismometers are insensitive to rotary motion if the output signal is referred to the location of the point mass of the equivalent simple pendulum. The observation of rotary motion deserves specific sensors that are insensitive to linear acceleration.

Rigid-body motion

Seismometers are intended to record the displacement of the ground. The output signal of the seismometer thus must be attributed to the motion of one point. The choice of this location is arbitrary in cases where a suspended body is used. For a pendulum the displacement of each location on the seismometer’s frame is the same as well as the ground’s displacement (Fig. 1). In the presence of rotations this is no longer the case. Each point on the extended body undergoes a different displacement while the rotation is the same for all (Fig. 2). Then a reference location must be defined to which the output signal is attributed. This reference location need not be the hinge of the pendulum nor needs it be the location of the center of mass, necessarily.

Reference location

In this study I discuss the output signal of a pendulum seismometer in response to the motion of its frame. I use a simple model of the seismometer consisting of a casing (green rectangle in Fig. 3), a pendulum (blue body) with its center of mass at S and being attached to the casing by a hinge at H. I study the response of the seismometer to a motion of the reference location R on the seismometer’s frame.

Equation of motion

The geometrical quantities used for the mathematical treatment are defined in Fig. 5. The Lagrangian of the seismometer’s pendulum is

\[ L = \frac{1}{2} (\dot{y} + \dot{y}_0)^2 + \frac{1}{2} m l^2 \dot{\phi}^2 \]

where \( y \) and \( y_0 \) are components of the location vector

\[ \mathbf{y}(t) = (y(t), y_0(t), \phi(t)) \]

to the center of mass at S and \( R \) is the location to the reference location R to which the output signal is attributed. Lagrange’s equation results in the equation of motion

\[ m (\ddot{y} + \dot{y}_0 \dot{\phi}) + m l \dot{\phi} \cos \phi + m l^2 \dot{\phi}^2 \sin \phi = 0 \]

after a few pages of calculus. With

\[ \beta = \phi - \alpha, \quad \dot{\phi}_1 = \cos \beta \]

and \( \dot{\phi}_2 = \sin \beta \)

I obtain

\[ m \ddot{y} + m l \ddot{\phi} \cos \phi + m l^2 \dot{\phi}^2 \sin \beta = 0 \]

(7)

where \( \beta \) is the quantity observed by the transducer.

Sensitivity

I understand the pendulum to be held in its reference position by a feedback such that \( \alpha \) and \( \dot{\alpha} \) are constant. By solving eq. (7) for \( \beta \) I derive the expression

\[ \beta = -\frac{1}{m l} \left( \dot{\phi}_1 + \dot{\phi}_2 \right) \]

for the sensitivity of the pendulum. The three terms on the right-hand side of eq. (8) are the contributions for three types of motion.

- **translational acceleration** \( \beta = \frac{l}{m} \left( \dot{\phi}_1 + \dot{\phi}_2 \right) \)
- **angular acceleration** \( \beta = -\frac{1}{m l} \left( \dot{\phi}_1 + \dot{\phi}_2 \right) \)
- **centripetal acceleration** \( \beta = -\frac{1}{m l} \left( \dot{\phi}_1 + \dot{\phi}_2 \right) \)

Conclusions

If the output signal is referred to the location of the point mass of the equivalent simple pendulum, i.e. \( R \) is chosen such that \( \alpha = -\dot{\alpha} = 0 \), then

\[ \beta = -\frac{1}{m l} \left( \dot{\phi}_1 + \dot{\phi}_2 \right) \]

for any motion. Hence the seismometer is only sensitive to the translational acceleration of the location of the point mass of the equivalent simple pendulum. If the output signal is referred to the location of the point mass of the equivalent simple pendulum, i.e. \( R \) is chosen such that \( \alpha = -\dot{\alpha} = 0 \), then

\[ \beta = -\frac{1}{m l} \left( \dot{\phi}_1 + \dot{\phi}_2 \right) \]

For a sensor for rotary motion it can be constructed such that \( H \) and \( S \) coincide. Then always

\[ \beta = -\frac{1}{m l} \left( \dot{\phi}_1 + \dot{\phi}_2 \right) \]

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References

