Geophys. J. Int. (1992) **111,** 226–236

Observation of low-order toroidal modes from the 1989 Macquarie Rise event

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Accepted 1992 April 23. Received 1992 April 14; in original form 1991 November 18

SUMMARY

At frequencies below 1 mHz toroidal modes are more difficult to observe than spheroidal modes for two reasons. First, the comparatively rapid decay of the signal (low Q factor) of toroidal modes and the consequently short records which have to be analysed result in low resolution in frequency. Secondly, high noise on horizontal component seismometers due to their sensitivity to tilts induced by atmospheric disturbances makes it difficult to observe long-period motion. Here we present observations of the lowest order toroidal modes based largely on digital recordings of the 1989 Macquarie Rise event. We present what we believe to be the first unambiguous observation of the gravest toroidal mode ${}_0T_2$. We estimate multiplet degenerate frequencies and Q for the fundamental modes up to ${}_0T_{10}$ and the overtone ${}_1T_6$. The frequencies of individual singlets are estimated for the modes ${}_0T_5$–${}_0T_8$, and for the three multiplets ${}_0T_8$–${}_0T_{10}$ we have sufficient signal to estimate also the degree 2 aspherical structure coefficients.

Key words: earthquake source mechanism, Macquarie Rise earthquake, strainmeter, toroidal free oscillations.

1 INTRODUCTION

Numerous recent deployments of three-component seismometers with improved sensitivity at the low-frequency end of the normal mode spectrum have made the Macquarie Rise event (1989 May 23, Moment: 2×10^{21} Nm) a prime candidate for the study of low-order toroidal modes. Observations of the degenerate frequency of these modes allows us to constrain average mantle structure. Furthermore, the modes exhibit some sensitivity to structure of the core–mantle boundary and one can, in principle, use information about the splitting of these modes to constrain the location of the structure responsible for anomalous splitting (Widmer, Masters & Gilbert 1990).

We inspected all digital long-period (LP) and very long-period (VLP) recordings from the major global networks (IDA, SRO, ASRO, GEOSCOPE, upgraded DWWSSN, CDSN, IRIS) and the Black Forest Observatory (BFO) in SW Germany. We retained 116 records with a high signal-to-noise ratio for the study of low-order toroidal modes and the determination of the source mechanism. When added to our data base of long-period seismograms of the largest earthquakes since 1977, the recordings of the Macquarie Rise event more than double the number of traces which contain signal for the fundamental toroidal modes below 1 mHz. The large number of clean recordings available for this earthquake shows how much the digital networks have expanded since 1977 when source mechanisms were frequently constrained by data from less than 10 stations. Forty recordings were discarded, sometimes due to obvious problems with either the seismic sensor or the data logger but, in the majority of cases, due to environmental noise at the recording site.

Atmospheric conditions at the recording site are of prime importance for the observation of free oscillations with periods exceeding 1000 s. This is illustrated by the observation that seismic noise at frequencies below 1 mHz due to the passage of a cold front can easily swamp the signal of all but the largest earthquakes (Sorrells 1971; Müller & Zürn 1983). Therefore shielding the instruments as much as possible from the direct influence of atmospheric conditions is of prime importance. The circumstance that very stable weather conditions existed in Central Europe for the days following the Macquarie Rise event is at least partially responsible for the high quality of the BFO records.

In Section 2 we perform a moment tensor inversion in the

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frequency band 2–6 mHz and our results confirm the strike–slip nature of the event found by earlier studies (Romanowicz & Ekström 1989). In fact, the Macquarie Rise event is the largest strike–slip event recorded with modern instruments and its source mechanism is ideal for the excitation of toroidal modes. We have been able to observe fundamental toroidal modes on amplitude spectra of single records down to ${}_0T_2$.

Since all previous reported observations of the gravest toroidal mode ${}_0T_2$ are derived from recordings of the 1960 Chilean earthquake, we were curious to see if we can confirm these observations. In Section 3 we reanalyse the Isabella strainmeter recorder of the 1960 Chilean event (Benioff, Press & Smith 1961) which, according to Derr (1969), has yielded the best spectra of ${}_0T_2$. This record is also the only recording of the Chilean event which is available to us in digital form. Our search for ${}_0T_2$ in the Isabella strainmeter record using the multitaper spectral analysis technique (Park, Lindberg & Thomson 1987; Lindberg & Park 1987) is unsuccessful and we believe that the Macquarie Rise event has led to the first unambiguous observation of this mode. In Section 3.3 we employ the BFO strainmeter array to demonstrate the ${}_0T_2$ observation more clearly.

We use two techniques to estimate multiplet degenerate frequencies and Q of the low-order fundamental toroidal modes. In Section 4 we analyse peak frequency measurements from individual records and in Section 5 we perform singlet stripping (e.g. Ritzwoller, Masters & Gilbert 1986). The latter technique is a phase equalization procedure which permits simultaneous use of multiple records from multiple events and is particularly suited for low l multiplets.

Coriolis coupling between fundamental toroidal and spheroidal modes is strongest for source–receiver configurations with near-polar great circles (Park 1986). It is therefore understandable why the 1981 May 25 Macquarie Rise event, located at 52°S, has led to the first observations of Coriolis coupling (Masters, Park & Gilbert 1983). The 1989 Macquarie Rise event also resulted in numerous recordings corresponding to near-polar great circles and, looking at amplitude spectra of such recordings, we find again ample evidence for Coriolis coupling. We note, however that, among the modes of interest in this paper, only ${}_0T_9$ and ${}_0T_{10}$ are noticeably affected.

2 MOMENT TENSOR INVERSION

We adopt the model of a point source in space with a triangular source time function to describe the earthquake rupture. We leave the source location and the event time fixed at the values determined by NEIC (1989); event time: 1989 May 23 10 h 54:49.5 UT; latitude: 52.34°S; longitude: 160.57°E; depth: 10 km. A quantitative description of the source is needed as part of the singlet stripping technique so that the excitation of individual singlets can be calculated. To justify the use of the point source model in singlet stripping experiments for the modes ${}_0T_5$–${}_0T_8$, we show that higher order terms in the moment tensor expansion which account for the spatial extent of the source are negligible compared with the zeroth-order moment tensor describing a point source. Backus & Mulcahy (1976) have shown that the magnitude of the higher order terms in the Taylor series expansion of the displacement field in the source volume falls off as $(1/n!)(6\pi\ell/\lambda)^n$ where ℓ is the largest distance of any point in the source volume from the origin and λ is the smallest wavelength of interest. In our case, we find that the mode ${}_0T_8$ has a wavelength $\lambda \simeq 2\pi a/(l+1/2) = 4700$ km, where a is the radius of the Earth and $l = 8$ is the harmonic degree. The fault length of the Macquarie Rise event has been estimated to be $L \simeq 90$ km based on a study of broad-band body waves and the distribution of aftershocks (Braunmiller & Nábělek 1990). If we assume that our origin is centred on the fault we have $\ell = L/2$ and we get a ratio of the second term over the leading term of $(1/2)(6\pi\ell/\lambda) = 0.09$. This error is acceptable for our purpose since, in practice, all one has to do for singlet stripping to succeed is to predict correctly the sign of the initial motion of the target singlet at the receiver. Models which take the spatial extent of earthquake ruptures into account have been put forward by Backus & Mulcahy (1976) and Backus (1977) and, in the case of the Macquarie Rise event, their use is mandated for studies of all but the modes with the lowest harmonic degree.

We calculate the Green's functions associated with the six independent elements of the zeroth-order moment rate tensor by modal summation. For the well-studied fundamental spheroidal and toroidal modes, we also account for the zeroth-order effect of aspherical structure and shift the centre frequencies to match the observed great circle average (Jordan 1978; Smith & Masters 1989a). The moment tensor inversion is performed by linear fitting in the frequency domain and the rise time is varied until an optimum fit to the data is obtained. Among the 214 events in our data base we have not found a single event where the fit to the data was significantly improved by allowing the moment tensor to have an isotropic component. We also found this to be true for the 1989 Macquarie Rise event and we therefore remove this degree of freedom and constrain the moment tensor to be deviatoric. Decomposing the resulting moment tensor into major and minor double-couples (Gilbert 1980) we find that the two differ by more than a factor 20 in size and can therefore conclude that a single double-couple is an adequate representation of the source. Linear fitting tends to give an estimate of moment which is biased low because of a misalignment of the Green's functions with respect to the data caused by aspherical structure. We correct for this effect by rotating and scaling the double-couple until the source best predicts the observed power at each station. Since noise is included in the power estimate, this latter technique tends to overestimate the true moment of the event. Our estimates of the scalar moment m_0 lie between $16 \pm 4 \times 10^{20}$ Nm. The strike of the fault plane is $\sigma = 35° \pm 1°$, the dip is $\delta = 78° \pm 2°$ and the rake is $\gamma = 185° \pm 30°$. For the auxiliary fault plane we get $\sigma = 126° \pm 1°$, $\delta = 83° \pm 2°$ and $\gamma = 349° \pm 30°$. The errors in the fault plane orientation have been estimated from the errors on the moment tensor elements using a bootstrapping technique (Efron & Tibshirani 1986) and by comparing solutions obtained from different subsets of the data. The rise time for this event is well determined and is 45 s which is unusually short for this size of event. A comparison with the tectonic setting in which the event occurred suggests that a right lateral rupture occurred on the NE–SW trending fault plane (Ekström & Romanowicz 1990).

For the solution presented above we have used $J = 116$ seismic traces: 60 vertical and 56 horizontal component recordings. As a measure of goodness of fit we use the mean variance reduction. Let the jth trace be x_{ij}, $i = 1, \ldots, N_j$ and the synthetic seismogram computed with the final source mechanism be y_{ij}, $i = 1, \ldots, N_j$, then we define the mean variance reduction for the moment tensor solution as

$$V_{\text{red}} = \frac{100}{J} \sum_{j=1}^{J} \left[1 - \frac{\sum_{i=1}^{N_j} (x_{ij} - y_{ij})^2}{\sum_{i=1}^{N_j} x_{ij}^2} \right] \quad (1)$$

For the Macquarie Rise event we obtain a mean variance reduction of only $V_{\text{red}} = 43$ per cent. The cause for this poor fit could be due to three reasons: (1) instrument calibration errors, (2) inadequately modelled 3-D structure, and (3) the large spatial extent of the source. Instrument calibration errors can be largely ruled out since we were able to obtain a source mechanism with 77 per cent variance reduction using 66 seismograms for an intermediate size, deep earthquake only three weeks prior to the Macquarie Rise event (Peruvian earthquake: 1989 May 5, $m_0 = 0.5 \times 10^{20}$ Nm, depth: 593 km). The second and third possible reasons are probably equally responsible for the poor variance reduction. It has been our experience that shallow and intermediate depth earthquakes are more difficult to model than deep focus events (Masters, Priestley & Gilbert 1983). The Macquarie Rise event is shallow and so the low variance reduction falls within the known pattern. Oddly enough, we still lack a plausible explanation as to why spectra from recordings of deep earthquakes can be modelled better with spherical Earth Green's functions than spectra from shallow events. We think this observation is odd because, between 2 and 6 mHz, spectra from 10 h long time series (such as those generally used in source retrieval) are dominated by fundamental mode peaks independent of the source depth.

3 THE SEARCH FOR ${}_0T_2$

A systematic search for the excitation of the gravest toroidal mode must concentrate on recordings of the largest events in the last three decades. Earlier events have to be discarded because of the lack of recordings with sufficient dynamic range. The records must be either from a horizontal strainmeter or a horizontal seismometer since toroidal motion does not involve any vertical displacement. The largest earthquakes in this time window are, in chronological order, the 1960 Chilean event, the 1964 Alaskan event, the 1977 Indonesian event, the 1987 Alaskan event, and the 1989 Macquarie Rise event.

For the first two events, the best horizontal recordings made with instruments whose passband also contains the frequency of ${}_0T_2$ are the Isabella strainmeter records (Benioff *et al.* 1961) and the Trieste tilt meter record (Bolt & Marussi 1962). All the three-component seismometers that were deployed following the International Geophysical Year (1957/58) had a passband too narrow to record ${}_0T_2$. Starting in 1969, the first well-calibrated, three-component HGLP seismometers were installed. These early instruments together with the subsequent SRO sensors had a rapid fall-off at the low-frequency end of the passband. Even so, the best horizontal recording of the 1977 Indonesian event by an ASRO (a modified HGLP) instrument (ZOBO, E–W) shows a clear peak at the frequency of ${}_0T_4$ and to a lesser extent at the frequency of ${}_0T_3$. No high-quality strainmeter recordings of the 1977 Indonesian event are available.

In the last decade, major improvements in the design of both the seismic sensors (Wielandt & Streckeisen 1982) and the digital data loggers (e.g. Wielandt & Steim 1986) were achieved so that it became possible to record the entire seismic spectrum relevant to earthquake seismology (from 0.3 mHz to 20 Hz) with a dynamic range of 140 dB using a single data stream. The wide acceptance of this new generation of hardware has led to numerous high-quality, three-component recordings of both the 1987 Alaskan and 1989 Macquarie Rise event. In the case of ${}_0T_2$ however we have found that the only instrument which recorded that mode was the BFO invar strainmeter array described in the next section.

3.1 The 120° strainmeter array at Schiltach Observatory (BFO)

A general description of the mine observatory can be found in Mälzer *et al.* (1979) and Emter & Zürn (1985). The strainmeter array is installed at the very end of the tunnel system under about 170 m of overburden and a horizontal distance of 650 m from the mine entrance. An airlock reduces air pressure variations near the instruments and temperature variations are in the order of 1 mK. Three tunnels at angles of 120° and 60 m long are the sites of three strainmeters with azimuths of N2°E, N120°E and N60°W. The strainmeters are slightly modified versions of the invar wire devices described by King & Bilham (1976) and Agnew (1986) and have a length of 10 m. One modification is the inclusion of an *in situ* calibration device in the form of an expandable bearing plate. The output signals from the inductive displacement transducers (LVDTs) are fed into free mode bandpass filters with amplifications of about 600 and cut-off periods of about 80 and 3000 s, respectively. The output signals of these filters are then anti-alias filtered with a cut-off period of 25 s and digitized in intervals of 2 s and a resolution of 16 bits. The least significant bit in the passband corresponds to approximately 0.3×10^{-12} of strain, and because of the design it corresponds to a displacement of 0.03×10^{-9} m at the LVDTs (length of 10 m, mechanical amplification of 10 by lever arms).

3.2 Application of the multitaper technique to individual records

To search for the presence of the mode ${}_0T_2$ in a time series, we use the multitaper spectral analysis technique developed by Park *et al.* (1987) and Lindberg & Park (1987). This non-parametric technique is the method of choice as it has been demonstrated to possess superior detection capabilities (Thomson 1982) over single taper estimates and over parametric techniques.

An important difficulty with detecting a small-amplitude harmonic component in a time series of finite length is spectral leakage. The multitaper technique explicitly addresses this point. A set of orthogonal tapers are designed to reduce optimally the spectral leakage of a decaying cosinusoid immersed in white noise. This optimization problem can be cast in terms of a generalized matrix eigenvalue problem with the eigenvectors being the optimal tapers and the eigenvalues being a measure of the efficiency of the tapers to suppress spectral leakage. The tapers are designed to take into account the decay rate of the target mode as well as the variance of the noise and the data gaps of the particular record to be analysed.

We select the K tapers with the best leakage reduction properties. We multiply the seismogram to be analysed by these K tapers which results in an equal number of time series. In a least-squares procedure, we fit a decaying cosinusoid multiplied by the kth taper to the kth time series. In this way, we obtain K uncorrelated amplitude estimates at every frequency from just one seismogram. We then average the K estimates to obtain a more accurate amplitude estimate than would have been possible with just a single (e.g., Hanning) taper estimate. A statistical F-test is performed to test the null hypothesis that the initial amplitude of the decaying cosinusoid vanishes. Conversely, one can use the F-test to measure the likelihood of the presence of a decaying cosinusoid at a given frequency.

In order to compute the optimal tapers, we need an *a priori* estimate of the decay rate of the target multiplet and the variance of the noise present in the seismic record. The Q value of ${}_0T_2$ can be predicted accurately since this mode samples all parts of the mantle and the average Q value of the mantle is the best constrained property of Q models (Masters & Gilbert 1983; Widmer, Masters & Gilbert 1991). Rather than estimating the noise variance v, we choose to experiment with a range of v values and base our choice of the final v on a visual assessment of the estimated amplitude spectra.

Fig. 1 illustrates the results of the multitaper procedure applied to the 450 hr long Isabella strainmeter record of the 1960 Chilean earthquake. In Fig. 2, the multitaper results are summarized for the much shorter BFO strainmeter record of the Macquarie Rise event. We also used a more conventional method to show the presence of a coherent spectral line at the fundamental mode frequencies by plotting amplitude spectra of variable length, Hanning tapered, data windows.

Before we interpret the F-test value plots (middle panel of Figs 1 and 2) it is helpful to estimate the expected number of false positives. For the Isabella (BFO) strainmeter record with an effective length of 250 (80) hr we have 818 (260) independent frequency estimates in the frequency band shown in the figures (0.1–1.0 mHz). If the time series consisted of Gaussian noise we would expect the F-test value to exceed the 95 per cent (99 per cent) level in 5 (1) out of 100 F-test estimates. Thus we expect 41 (8) false positives at the 95 per cent (99 per cent) level for the Isabella strainmeter record and only 13 (2.6) false positives for the BFO strainmeter record at the 95 per cent (99 per cent) level.

While we are unable to detect ${}_0T_2$ in the Isabella strainmeter record with either method, our tests for the presence of a decaying sinusoid in the vicinity of the predicted frequency of ${}_0T_2$ is successful and exceeds the 99 per cent confidence level for the BFO invar strainmeter record of the Macquarie Rise event. Assuming that our copy of the Isabella strainmeter record is a true replica of the original time series and has survived 30 yr of storage on various computer media unchanged, we conclude that previous claims of ${}_0T_2$ observations were associated with spurious peaks. (The fact that we can see the $m = \pm 1$ singlets of ${}_0S_2$ as well as ${}_0S_0$ in spectra of the Isabella record gives us confidence that we have a true replica of the original record).

To measure actually the degenerate frequency of ${}_0T_2$, we first note that a double-couple source excites the $\pm m$ singlets with equal amplitude. Given the short record length of the BFO strainmeter record, we are unable to resolve the splitting of ${}_0T_2$. Since the splitting of this mode is predicted to be entirely dominated by rotation and is thus symmetric about the centre frequency, we can expect to get an unbiased estimate of the centre frequency if we fit a single resonance function to the data spectrum. The centre frequency estimate for ${}_0T_2$ given in Table 1 is based on this technique and the only series used are the N2°E and the N300°E BFO invar strainmeter records.

3.3 Observations based on the BFO strainmeter array

Neglecting local distortions of the strain field (e.g. Emter & Zürn 1985), three non-colinear strainmeters can be used to determine the three horizontal components of the strain tensor. The output of a linear strainmeter is given by

$$\frac{\delta l}{l_0}(\psi) = \epsilon_{\theta\theta} \cdot \cos^2(\psi) + \epsilon_{\phi\phi} \cdot \sin^2(\psi) - \epsilon_{\theta\phi} \cdot \sin(2\psi), \quad (2)$$

where ϵ_{ij} are the components of the strain tensor in a coordinate system with the θ axis pointing south and the ϕ axis pointing east and ψ is the azimuth of the strainmeter measured clockwise from north. δl is the change in the length and l_0 the length of the device. From (2) it follows that the calibrated sum of the signals from three strainmeters with azimuths differing mutually by 120° is 1.5 times the areal strain. Since the areal strain of torsional modes is predicted to be identically zero, the spectrum of such a locally stacked signal should be devoid of torsional peaks. The upper panel of Fig. 3 shows the spectrum of the areal strain at BFO after the Macquarie Rise event. Clearly the spectrum is dominated by spheroidal modes. Alternatively, the three strainmeter signals can be linearly combined to enhance the shear strain at a particular frequency. Fig. 3 (lower panel) shows the strain spectrum for the azimuth $\psi = \psi_{\max}$ for which the amplitude of ${}_0T_2$ is maximal. Note that all the fundamental toroidal modes in this spectrum are present well above the noise. It turns out that the direction $\psi_{\max}$ points from the station to the epicentre. The amplitude of ${}_0T_2$ about 2 hr after origin time was determined from the spectra to be about 10×10^{-12} of strain. Based on the moment tensor determined in Section 2 the predicted amplitude of ${}_0T_2$ is 6×10^{-12}. This compares well with the observed values and shows that the amplitude of ${}_0T_2$ was sufficiently large to be recorded by the strainmeters.

4 MODES ${}_0T_3$–${}_0T_{10}$

As the number of records available per mode increases, we can use techniques for the estimation of the multiplet

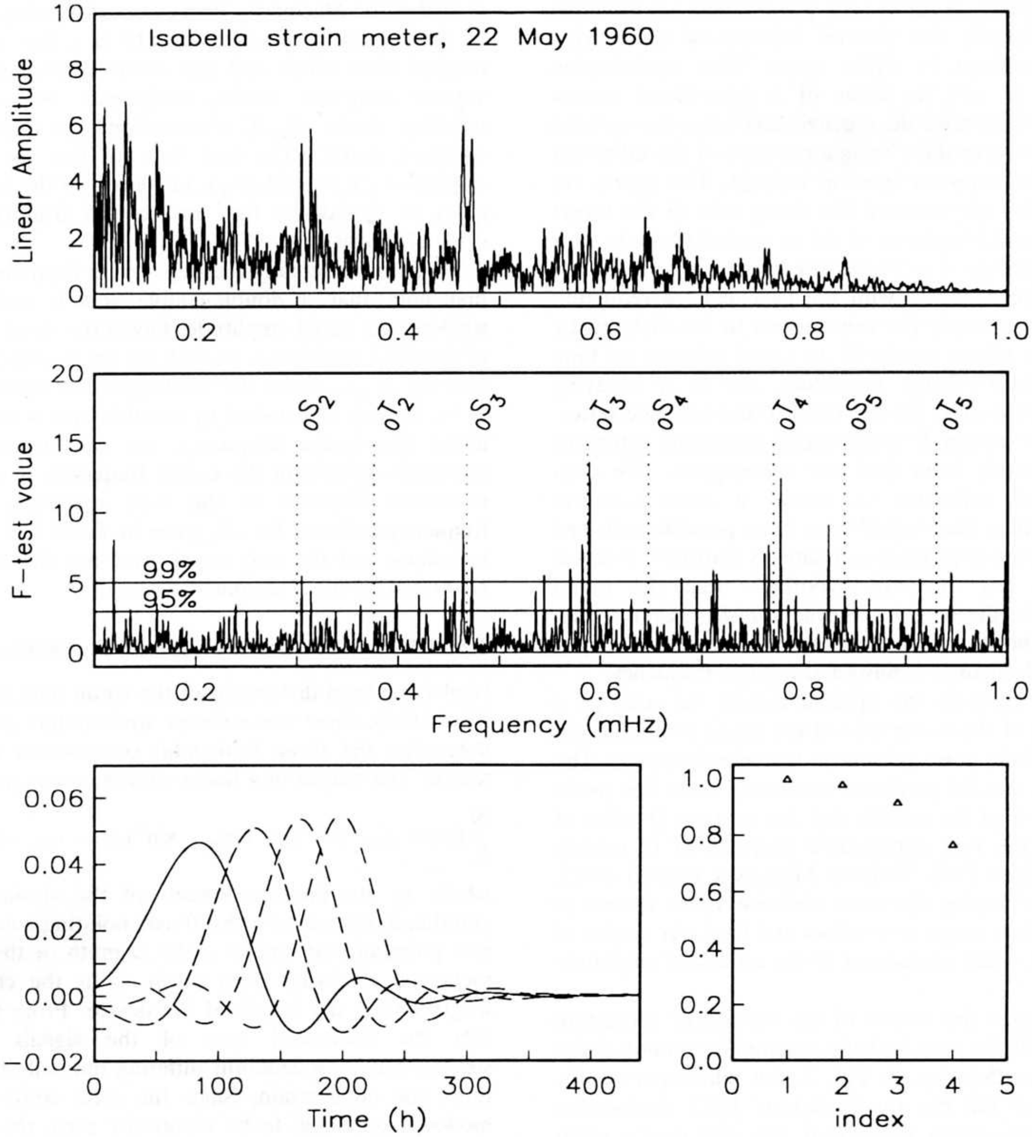

Figure 1. Multitaper spectral analysis of the Isabella strainmeter record of the 1960 Chilean earthquake. The multitapers are optimized for the detection of ${}_0T_2$. The top panel shows the estimated amplitude spectrum with ${}_0S_2$ and ${}_0S_3$ forming the largest spectral peaks in the mode band. Both these multiplets are visibly split due to the Earth's rotation. The middle panel shows the results of the variance ratio F-test. The vertical dashed lines indicate the location of the fundamental mode degenerate frequencies for model 1066A. The two solid horizontal lines indicate the 95 and 99 per cent confidence levels for the presence of a decaying cosinusoid with decay rate $\alpha = 4.3 \times 10^{-6}$ rad s^{-1}. While the test is successful for the multiplets ${}_0S_3$, ${}_0T_3$ and ${}_0T_4$, it clearly fails for ${}_0T_2$. The bottom left panel shows the four lowest order eigentapers which were used (design parameters: $\Omega = 8\pi/N$, $v = 0.01$). The tapers only sample the first half of the record because of the relatively low-quality factor predicted for ${}_0T_2$ ($Q = 275$). The efficiency of the tapers to suppress spectral leakage is measured by the so-called bandwidth retention factors shown in the last panel. Only the first four eigentapers were used because the bandwidth retention factor is less than 0.5 for the fifth and all higher order tapers.

degenerate frequency which make less restrictive assumptions about the nature of the large-scale aspherical structure.

The 'diagonal sum rule' (Gilbert 1971) states that the average value of the singlet frequencies of a multiplet split by linearly small aspherical perturbations is the multiplet's degenerate frequency. Given that the large-scale aspherical perturbations are small and given a set of records which sample the Earth evenly, the diagonal sum rule implies that peak frequency measurements made from these records average to the degenerate frequency of the multiplet. While the first condition is generally met, the second one is not. A plot of the distribution of the great circle poles associated with the seismograms of our data set shows that the sampling of the globe is very uneven. Hence we cannot blindly apply the diagonal sum rule and average the peak frequency measurements to obtain the degenerate frequency. A further complication with the use of the diagonal sum rule occurs at frequencies below 1 mHz where the second-order effect of rotation becomes significant, and between 1 and 4 mHz where Coriolis coupling between nearby fundamental spheroidal and toroidal modes has been observed. Both of these effects perturb the mean of the

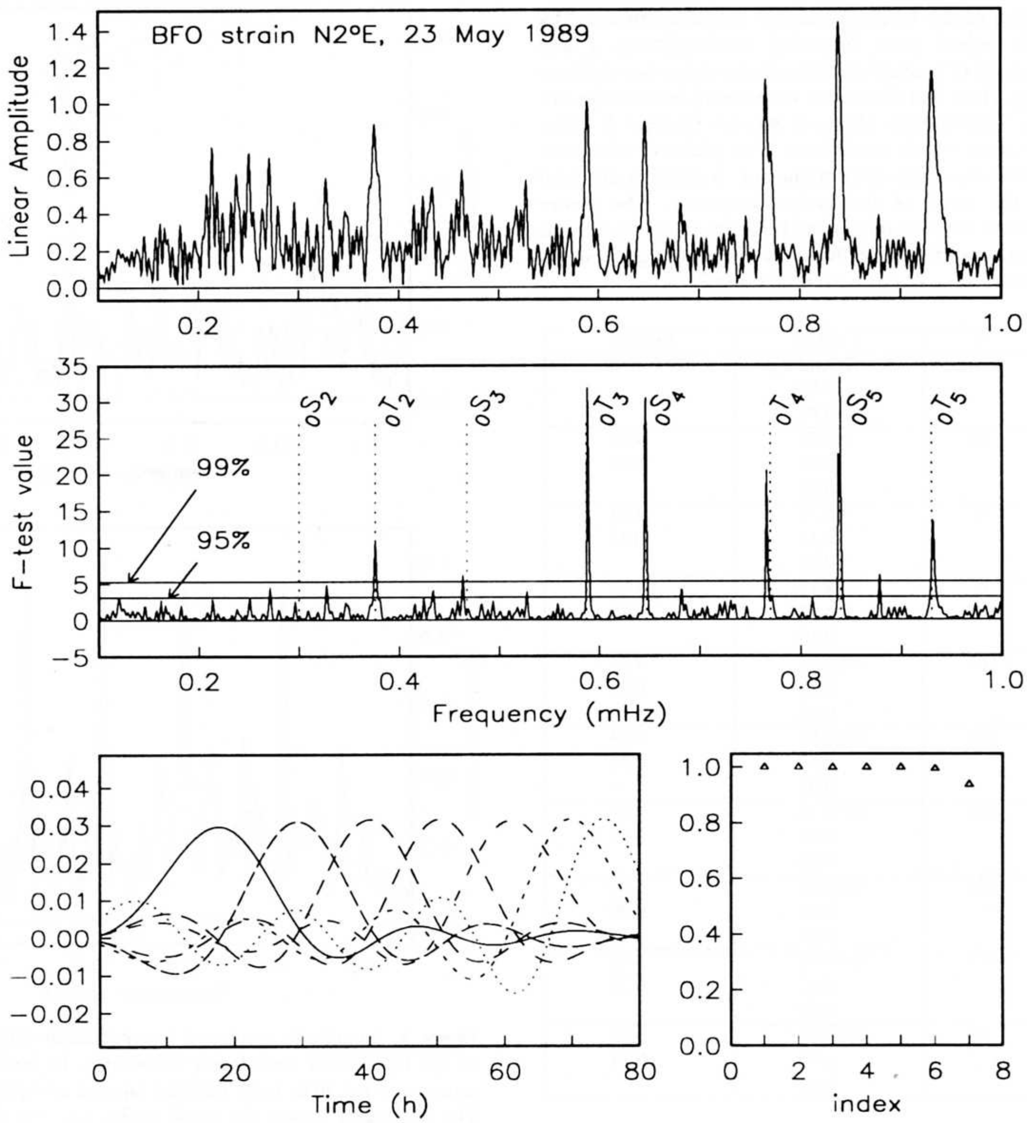

Figure 2. Multitaper spectral analysis of the BFO strainmeter record of the Macquarie Rise earthquake. The multitapers are optimized for the detection of ${}_0T_2$. The top panel shows the estimated amplitude spectrum with ${}_0T_2$, ${}_0T_3$, ${}_0S_4$, ${}_0T_4$ and ${}_0S_5$ forming the largest spectral peaks in the mode band. Mode splitting due to rotation is not visible due to the short record length. The middle panel shows the results of the variance ratio F-test. The test fails to detect only ${}_0S_2$. All other fundamental modes have F-test values exceeding the 99 per cent level. The F-test peak for ${}_0S_5$ is clipped and reaches 65. The bottom left panel shows the seven lowest order eigentapers which were used (design parameters: $\Omega = 8\pi/N$, $v = 0.01$). The tapers sample all parts of the record because the amplitude of ${}_0T_2$ is predicted to decay by only 18 per cent over an 80 hr period. The bandwidth retention factors for the first seven eigentapers are shown in the last panel.

singlet frequencies away from the degenerate frequency and must therefore be corrected for. Given a reference model such as model 1066A (Gilbert & Dziewonski 1975) however, these shifts can be accurately predicted for all the modes considered in this study (Dahlen & Sailor 1979; Dahlen 1969).

While we have only a small number of records from which we can measure ${}_0T_3$ and ${}_0T_4$ and this set of records samples the Earth unevenly, we can argue, as with ${}_0T_2$, that (i) the splitting for these modes is dominated by rotation (linear in m), (ii) a double-couple source excites $\pm m$ singlets with equal amplitude, and hence (iii) the arithmetic mean of the peak frequency measurements is, apart from the second-order effect of rotation, an unbiased estimate of the multiplet degenerate frequency.

For the modes ${}_0T_5$–${}_0T_8$ the dominant splitting is due to the ellipticity of figure which manifests itself in a parabolic distribution of singlets. In this case a naïve application of the diagonal sum rule leads to multiplet degenerate frequency estimates biased towards low values since polar paths are generally underrepresented in our data set. As long as splitting due to ellipticity dominates over splitting due to general, non-axisymmetric aspherical structure, we can use the singlet stripping technique. This technique makes no implicit assumption about the frequency distribution of the singlets within one multiplet but assumes that the dominant aspherical structure sensed by a multiplet is axisymmetric (e.g. rotation and hydrostatic ellipticity). The application of this technique will be the subject of the next section.

Table 1. Multiplet centre frequency and Q estimates obtained by averaging single record peak frequency measurements. f and $1000/Q$ are averages of n observations and the errors are obtained by bootstrapping. Note that the centre frequencies in this table are not corrected for second-order effects of rotation (Dahlen & Sailor 1979). The correction, which would have to be added to the centre frequency to obtain the degenerate frequency, is given in the third row following the error of the centre frequency. The centre frequency estimates are also uncorrected for the effect of Coriolis coupling and the corrections for the three most strongly coupled modes in this table (${}_0T_8$–${}_0T_{10}$) are given in Table 2.

mode	n	f [μHz]	$1000/Q$
${}_0T_2$	2	376.60	–
		0.80	–
		0.67	
${}_0T_3$	15	587.26	4.50
		0.66	0.90
		0.33	
${}_0T_4$	35	766.73	3.77
		0.37	0.33
		0.18	
${}_0T_5$	39	928.97	4.92
		0.37	0.30
		0.10	
${}_0T_6$	163	1079.51	4.24
		0.18	0.27
		0.05	
${}_0T_7$	139	1221.50	4.84
		0.30	0.18
		0.01	
${}_0T_8$	193	1356.80	4.82
		0.25	0.16
		0.00	
${}_0T_9$	163	1487.56	5.00
		0.16	0.20
		0.00	
${}_0T_{10}$	182	1614.62	5.42
		0.17	0.26
		0.00	
${}_1T_6$	70	1925.51	3.00
		0.45	0.27
		0.00	

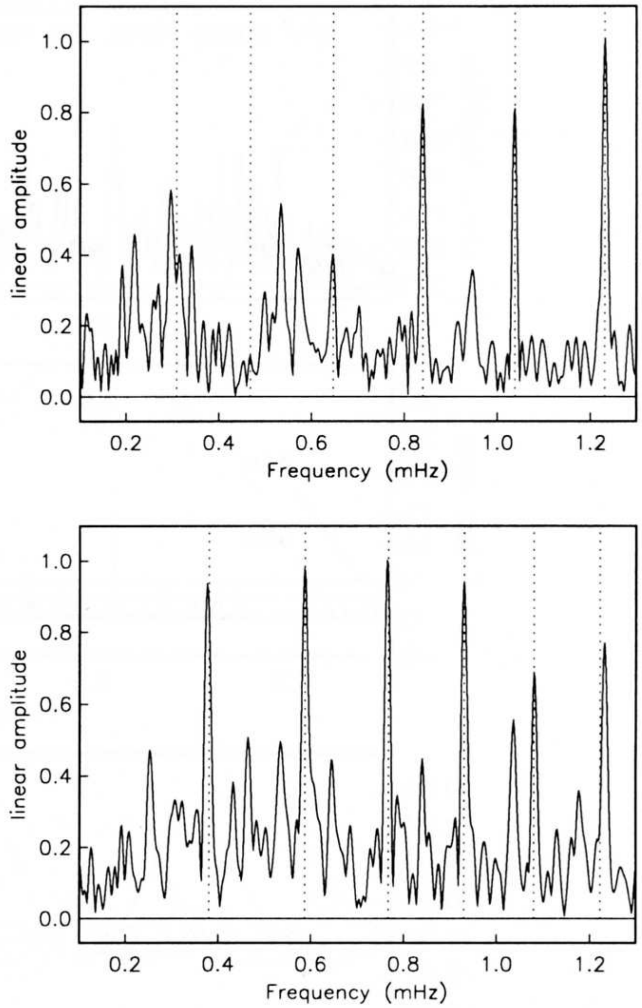

Figure 3. Amplitude spectra of two particular linear combinations of the three BFO strainmeter recordings. In both cases the time series used are 40 hr long, hanning tapered and padded with zeros. The top panel shows the areal strain, i.e. the dilatation in the horizontal plane. The vertical dashed lines indicate the location of the fundamental spheroidal modes ${}_0S_2$–${}_0S_7$ as predicted by model 1066A (Gilbert & Dziewonski 1975). The lower panel shows the maximal shear strain (see text). Here the vertical dashed lines indicate the predicted location of the fundamental toroidal modes ${}_0T_2$–${}_0T_7$. All the fundamental toroidal modes are well above the noise level. Note that, while the lower panel is dominated by the toroidal modes, they cancel completely in the upper panel. This observation confirms that the peaks in the lower panel are in fact associated with pure shear motion and are thus toroidal modes. The near-perfect cancellation of the toroidal modes in the upper panel can also be taken as a confirmation that the instrument calibrations are being modelled correctly.

For the modes ${}_0T_8$–${}_0T_{10}$ the effect of non-axisymmetric structure becomes so large that the singlet stripping technique starts to break down (e.g. the singlet strips have multiple peaks). However we have a sufficient number of individual peak frequency measurements for these multiplets, and their harmonic degree is sufficiently large that we can use Jordan's (1978) asymptotic theory and interpret the observed centre frequency shifts in terms of general aspherical structure along the great circle connecting source and receiver. The result of this analysis, the degenerate frequency and the degree $s = 2$ structure coefficients, are presented in Table 2. The dominant heterogeneity sensed by these modes is of degree $s = 2$ and in this case, the asymptotic theory works reasonably well for $l \geq 8$.

5 SINGLET STRIPPING FOR ${}_0T_5$–${}_0T_8$

The singlet stripping technique, which assumes that the aspherical structure sensed by a multiplet is dominantly axisymmetric, was first proposed by Gilbert (1971) and successfully applied to a number of spheroidal multiplets by Buland, Berger & Gilbert (1979), Masters & Gilbert (1981) and Ritzwoller *et al.* (1986). This technique allows us to estimate the singlet frequencies and from them, by use of the diagonal sum rule, the multiplet degenerate frequency. The distribution of the singlet frequencies can also be compared with the predictions for a rotating Earth model in hydrostatic equilibrium and inferences about non-hydrostatic, axisymmetric structure are possible. The goal of singlet stripping is to estimate the resonance functions of all singlets within a multiplet. We call these estimated

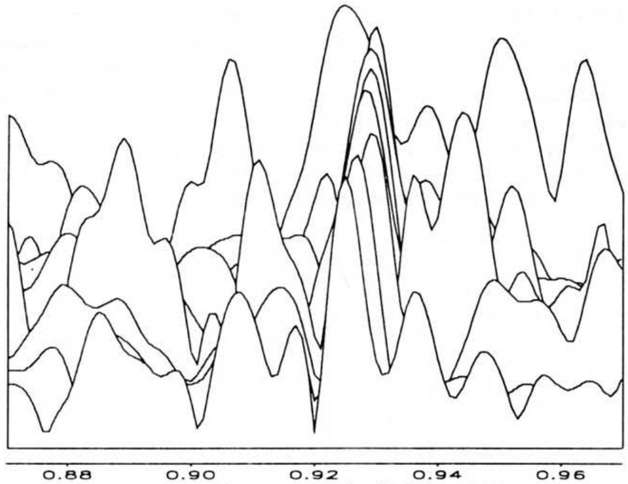

Figure 4. Amplitude spectra of the singlet strips for the multiplet ${}_0T_5$. The strips are arranged in ascending order with the one corresponding to the $m = -5$ singlet in front.

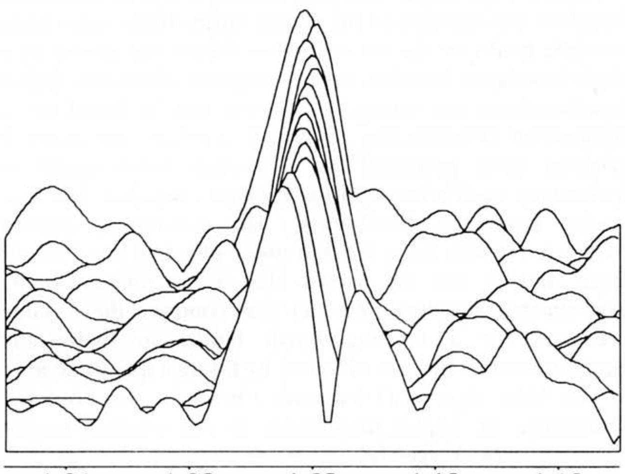

Figure 5. Amplitude spectra of the singlet strips for the multiplet ${}_0T_6$. The strips are arranged in ascending order with the one corresponding to the $m = -6$ singlet in front.

this signal is located remains unclear (Widmer *et al.* 1990). While axisymmetric anisotropy in the inner core may be physically acceptable, it cannot explain the splitting of several multiplets which have only very little sensitivity to inner core structure (e.g. ${}_{11}S_4$, ${}_{11}S_5$, ${}_{16}S_6$ and ${}_{21}S_6$). Degree 2 density or bulk modulus anomalies in the outer core can explain more of the observed signal though it is hard to see how such structures can persist in a convecting outer core. We are thus led to reconsider the CMB and the lowermost mantle as the location of the structure causing anomalous splitting. We therefore try to see if we can use the observed splitting of the fundamental toroidal modes to constrain the depth of the structure responsible for anomalous splitting.

In order to quantify the sensitivity of the low-order fundamental toroidal modes to structure at the base of the mantle we have calculated the effect of changing the ellipticity of the CMB on the c_2^0 structure coefficient. Doubling the ellipticity of the CMB leads to a 1.8 μHz perturbation in c_2^0 for ${}_0T_2$ but only a 0.3 μHz perturbation for ${}_0T_6$. This perturbation is much smaller than both the total c_2^0 coefficients estimated from the singlet frequency distribution (Table 2) and the associated errors. Similarly, if we introduce an axisymmetic degree $s = 2$ anomaly with an rms amplitude in shear wave velocity of 0.5 per cent located in D'' (the lowermost 200 km of the mantle), the c_2^0 of ${}_0T_2$ (${}_0T_6$) is only perturbed by 0.35 μHz (0.09 μHz). We therefore conclude that the toroidal mode observations presented in Tables 2 and 3 are not precise enough to place useful bounds on the location of the structure responsible for anomalous splitting.

Turning our attention to the problem of getting precise estimates of the complex multiplet degenerate frequency, we proceed by assuming that the dominant cause of splitting is rotation and hydrostatic ellipticity and that splitting parameters have the values predicted for model 1066A, and solve only for the degenerate frequency. The results of this fit are given in the second column of Table 3. The error estimates were obtained using a bootstrapping procedure (Efron & Tibshirani 1986). The degenerate frequencies for ${}_0T_5$–${}_0T_7$ obtained in this way agree only at the 3σ level with the values obtained by averaging individual peak frequency measurements (Table 1) and, for the reasons given above, we consider the values given in Table 3 to be the more precise estimates. For ${}_0T_8$ the degenerate frequency observations from all three techniques turn out to be consistent.

Q measurement of broadly split multiplets from single records exhibit large scatter due to the beating between the singlets. Singlet stripping turns out to be very successful in removing this source of scatter and we can reliably estimate the Q of the multiplet by averaging the Q values obtained from the singlet strips. The third column of Table 3 compares the observed Q values with the prediction of the Q model of Widmer *et al.* (1991).

6 CONCLUSION

With a scalar moment $m_0 = 16 \pm 4 \times 10^{20}$ Nm, the Macquarie Rise event was the largest earthquake of the last decade. The strike–slip source mechanism means that this event was very efficient at exciting toroidal free oscillations. We have used very long-period recordings of the horizontal ground acceleration and strain in the wake of this earthquake to study the lowest order fundamental toroidal modes. Using the singlet stripping technique, we have resolved the splitting of the modes ${}_0T_5$–${}_0T_8$ and estimated the singlet frequencies. The large number of peak frequency measurements available for the modes ${}_0T_8$–${}_0T_{10}$ has allowed us to estimate degree $s = 2$ aspherical structure coefficients for these modes. We have estimated the degenerate frequency for all fundamental toroidal modes up to ${}_0T_{10}$. Our observations of the modes ${}_0T_3$–${}_0T_9$ are consistent with earlier observations by Dziewonski & Gilbert (1972, 1973) and our estimates are between 1.5 and 4 times smaller. The large Coriolis correction for the mode ${}_0T_{10}$ makes our degenerate frequency estimate incompatible with earlier estimates. For ${}_0T_2$ our degenerate frequency estimate (uncorrected for second-order effects) is $376.6 \pm 0.8\,\mu$Hz

Table 2. Estimated degenerate frequencies and aspherical structure coefficients c_2^t in μHz for selected toroidal modes. To facilitate comparison with the results in Table 1 and 4 we did not correct the degenerate frequency and the c_2^0 coefficients for the effect of Coriolis coupling. For every mode we list in the first row the estimated coefficient followed by the estimated error. The third row contains the Coriolis correction which would have to be added to the values in the first row to get unbiased degenerate frequency and structure coefficient estimates (Smith & Masters 1989b).

mode	f [μHz]	c_2^0	Rec_2^1	Imc_2^1	Rec_2^2	Imc_2^2
${}_0T_8$	1356.56	−6.79	0.1	−2.1	-0.9	2.8
	0.25	1.50	0.7	1.1	0.7	0.9
	−0.12	−0.49	–	–	–	–
${}_0T_9$	1487.06	−5.26	1.2	−2.5	−0.2	3.6
	0.16	1.30	0.6	0.9	0.7	0.8
	−0.25	−0.96	–	–	–	–
${}_0T_{10}$	1613.46	−3.04	−1.0	−3.3	0.3	4.3
	0.17	1.30	0.6	0.9	0.7	0.8
	−0.50	−2.14	–	–	–	–

resonance functions 'singlet strips'. The singlet frequencies can subsequently be obtained by fitting a synthetic resonance function to the singlet strips (Masters & Gilbert 1983). Singlet stripping is a phase equalization technique and assumes that we know both the source mechanism and the shape of the singlet eigenfunctions. There exist several procedures to determine the low-frequency earthquake source mechanism (Gilbert & Dziewonski 1975; Dziewonski, Chou & Woodhouse 1981; Romanowicz & Guillemant 1984; Ekström 1989) and our own technique and the resulting solution for the Macquarie Rise event has been presented in Section 2.

Knowledge of the singlet eigenfunctions requires that we know the Earth's aspherical structure. This of course is what we would ultimately like to determine and is not known *a priori*. In the special case where the deviations from spherical symmetry are dominated by axisymmetric structure such as rotation and ellipticity of figure, the displacement eigenfunction of the mth singlet in the kth multiplet is known and makes singlet stripping possible. The ith observed spectrum $u_i(\omega)$, corrected for instrument response and source phase, is a weighted sum of singlet resonance functions $c_j(\omega)$ with the weights A_{ij} being determined by the source and receiver locations, the source mechanism and the singlet eigenfunctions

$$u_i(\omega) = A_{ij}(\omega)c_j(\omega). \tag{3}$$

Singlet stripping then requires solving (3) for the unknown resonance functions $c_j(\omega)$. The frequency dependence of the matrix $\mathbf{A}$ stems from the finite duration of the earthquake rupture. For a symmetric triangular source time function with the rise time τ as its base length, the frequency dependence of $\mathbf{A}$ is proportional to $f(\omega) = \mathrm{sinc}^2(\omega\tau/4)$ and can thus be considered to be constant over the frequency band of one multiplet.

The selection of the records which enter the left-hand side of (3) is crucial to the success of the method and we use only records whose amplitude spectra contain a clear peak at the frequency of the target multiplet. Besides the recordings of the Macquarie Rise event, we also include any records from our data base of digital long-period recordings of large earthquakes since 1977. From the 214 events in this data base, we find that the Indonesian event (1977 August 19) and the Alaskan event (1987 November 30) are the only other events to excite significantly toroidal modes below ${}_0T_6$.

Table 3. Frequency, Q and splitting parameters of low-order fundamental toroidal modes. For every mode the three lines contain (from top to bottom) the predictions for model 1066A, the observed values and their associated errors. The predicted rotational and elliptical splitting parameters are taken from Dahlen & Sailor (1979) and include all terms through second order in rotation and first order in ellipticity. The Q prediction in the third column are based on the Q model by Widmer *et al.* (1991).

mode	f [μHz]	$1000/Q$	a [10^{-3}]	b [10^{-3}]	c [10^{-3}]
	929.70	4.03	0.917	0.416	−0.102
${}_0T_5$	928.45	3.89	0.689	0.383	−0.068
	0.24	0.25	0.269	0.079	0.026
	1080.40	4.21	0.923	0.256	−0.069
${}_0T_6$	1078.83	4.01	0.796	0.241	−0.056
	0.18	0.37	0.210	0.052	0.015
	1222.36	4.41	0.926	0.170	−0.051
${}_0T_7$	1220.81	4.66	1.360	0.118	−0.072
	0.24	0.52	0.359	0.052	0.019
	1357.86	4.61	0.961	0.119	−0.038
${}_0T_8$	1356.79	4.95	1.140	0.089	−0.047
	0.25	0.33	0.277	0.041	0.011

Failure of the singlet stripping technique can be diagnosed based on the outcome of the singlet strips: if the strips have multiple peaks or do not otherwise follow the model of a single resonance function, our assumptions about the singlet eigenfunctions are wrong. A second test is based on a comparison between the rotational splitting parameter b (Dahlen 1968) predicted for a rotating Earth model in hydrostatic equilibrium and the b value estimated from the singlet frequency distribution. The rotational splitting parameter results from the first-order effect of the Coriolis force and it can be shown that it is unaffected by axisymmetric structure. We can thus compare the b value predicted for a rotating Earth model in hydrostatic equilibrium with the one obtained by fitting a quadratic in m of the form $\omega_m = \omega_0(1 + a + mb + m^2c)$ to the observed distribution of singlet frequencies to see whether singlet stripping was successful. The results of this fit are summarized in Table 3. The measured b values differ by less than one standard deviation from the values predicted for 1066A and are thus consistent with the working hypothesis of singlet stripping.

Figs 4 and 5 show perspective plots of the singlet strips of ${}_0T_5$ and ${}_0T_6$. The multiplets ${}_0T_6$ and ${}_0T_8$ are well isolated whereas ${}_0T_5$ is close to ${}_1S_3$ and ${}_0T_7$ is close to ${}_0S_7$. In any case, we do not expect these toroidal multiplets to be Coriolis-coupled since the harmonic degrees of nearby spheroidal modes do not differ by unity (Dahlen 1969). It is therefore consistent with our working hypothesis to strip simultaneously for all the singlets of the overlapping multiplets. The low signal-to-noise ratio for the singlet strips of ${}_0T_5$ is due both to the generally low signal-to-noise ratio in the original spectra and the small number of records available (see column 2 of Table 1). Attempts to strip for toroidal modes at frequencies lower than ${}_0T_5$ failed due to insufficient signal.

Several recent studies of resolvably split multiplets have found a large anomaly in the axisymmetric degree 2 aspherical structure coefficients (Ritzwoller *et al.* 1986, 1988; Giardini, Li & Woodhouse 1988; Li, Giardini & Woodhouse 1991) and the depth at which the structure responsible for compared with 379.0 μHz of Smith (1961), 379.3 μHz of Bolt & Currie (1975) and 378.7 ± 1.4 μHz given by Derr (1969) where this last value is the mean of seven separately published observations. Our attempt to confirm these earlier observations of the mode ${}_0T_2$ based on the Isabella strainmeter recording have failed and we believe that the BFO invar strainmeter recordings of the Macquarie Rise event have led to the first unambiguous observations of this mode. The precision of the degenerate frequency estimates presented here make it necessary to correct for second-order effects before they can be used as gross Earth data in an inversion for the spherically averaged Earth structure. We find no sign of anomalous splitting of the low-order toroidal modes but their sensitivity to aspherical structural perturbations near the base of the mantle is too weak to rule out the possibility (based on these observations) that the structure responsible for anomalous splitting is on the mantle side of the CMB.

In conclusion we note that a comparison of the signals from the three sets of horizontal instruments available at BFO (three component Streckeisen STS-1, N–S and E–W Askania pendula and the 120° invar strainmeter array) show that, for the observation of the lowest order toroidal modes, horizontal strain is at least a competitive signal to horizontal acceleration.

ACKNOWLEDGMENTS

D. Emter, W. Großmann, W. Kaminski and H. Otto contributed to the quality of the data from BFO. We also wish to thank the network operators of the IDA, GDSN, GEOSCOPE and IRIS networks for providing the research community with high quality, digital seismic data. The work has been supported by the National Science Foundation under grant EAR 90-04555 and the Deutsche Forschungsgemeinschaft through SFB 108, B5/D9.

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