Low−frequency limit for H/V studies due to tilt

Universität Karlsruhe (TH)
Research University · founded 1825

CRC 461
Strong Earthquakes:
A Challange for
Geosciences and
Civil Engineering

Thomas Forbriger

Geophysical Institute, University of Karlsruhe
and
Black Forest Observatory (BFO)
Research facility of the Universities of Karlsruhe and Stuttgart, Heubach 206, D−77709 Wolfach

1 Low-frequency H/V peak

Throughout the data recorded during the URS project (Ritter *et al.* 2005) in Bucharest city, we observe a low-frequency peak in H/V ratio near 0.2 Hz (Ziehm & Forbriger 2006). The frequency of this peak is systematically varying from lower frequencies in the north to higher frequencies in the south. This variation is believed to be caused by the dipping of the interface between the neogene and the cretaceous at depth from 1000 m to 2000 m. Unfortunately the H/V level below 0.15 Hz is unstable and strongly rises with decreasing frequency in the recordings of most stations. This puts an effective limit on the analysis of the peak at frequencies below 0.2 Hz (Fig. 1). Since in some of the data there appears to be a power-law relation between H/V ratio and frequency below 0.15 Hz, I discuss tilt as a possible cause of this strong increase.

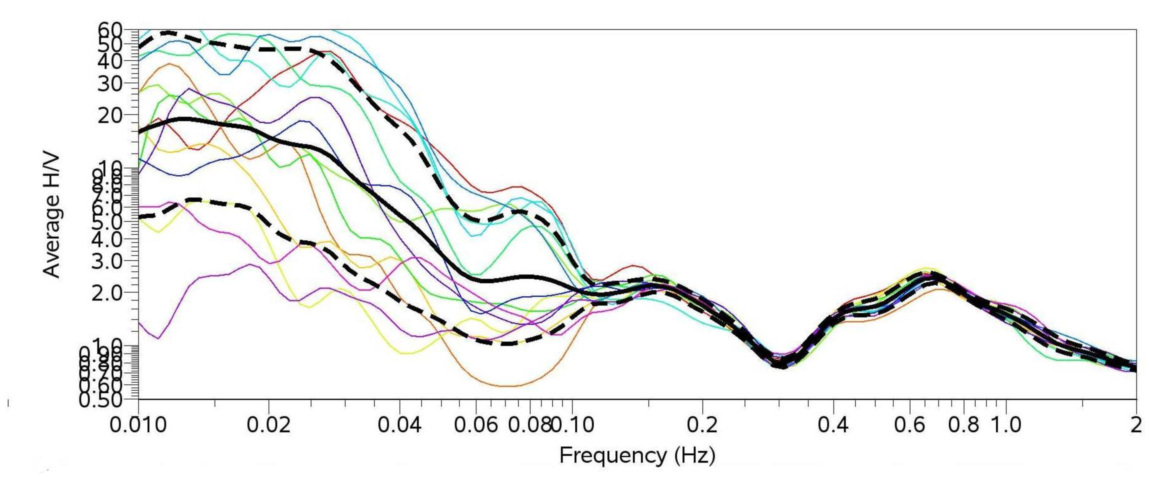

Figure 1: H/V ratio for station URS14. Two peaks (at 0.15 Hz and 0.7 Hz) are apparent. Each thin curve is H/V for a different time window, while the black curve displays the average with the dashed curves marking standard deviation. Courtesy of Julia Ziehm.

2 Forces acting on inertial sensors

Inertial sensors (gravimeters, seismometers and tiltmeters) sense inertial and gravitational acceleration and are principally unable to distinguish between both (Wielandt 2002, secs. 2.1 and 3.3). The inertial acceleration in the horizontal direction is

$$a_h = \frac{\partial^2 u_h}{\partial t^2} \quad \text{and} \quad a_z = \frac{\partial^2 u_z}{\partial t^2} \tag{1}$$

in vertical direction, where $u_z(h,z,t)$ is the vertical component of displacement and $u_h(h,z,t)$ the horizontal component. For small tilt angles the horizontal component of a seismometer sitting on the floor additionally senses the acceleration

$$a_\tau \approx g \frac{\partial u_z}{\partial h} \tag{2}$$

due to ground tilt, where g is the gravitational acceleration.

If $\tilde{a}_z$, $\tilde{a}_h$, and $\tilde{a}_\tau$ are the Fourier transforms of a_z, a_h, and a_τ, respectively, the Fourier transform of the seismometer's vertical component output signal is

$$\tilde{s}_z(\omega) = T(\omega)\tilde{a}_z(\omega), \tag{3}$$

where $T(\omega)$ is the instrument's complex response function. And

$$\tilde{s}_h(\omega) = T(\omega)\left(\tilde{a}_h(\omega) + \tilde{a}_\tau(\omega)\right) \tag{4}$$

for the horizontal component's output signal.

3 Tilt contribution to H/V ratio

H/V ratio is calculated from seismic recordings usually by

$$A_{\mathrm{H/V}}(\omega) = \frac{|R(\omega)\tilde{s}_h(\omega)|}{|R(\omega)\tilde{s}_z(\omega)|} = \frac{|\tilde{a}_h(\omega) + \tilde{a}_\tau(\omega)|}{|\tilde{a}_z(\omega)|} \tag{5}$$

and averaging over signals from several time windows. $R(\omega)$ is the response function of a deconvolution filter that provides a displacement signal from the electrical output signal $s(t)$. Hence the H/V ratio contains a tilt contribution, which may be ignored at high frequencies. At low frequencies the tilt contribution can dominate $A_{\mathrm{H/V}}$.

References

Ritter, J. R. R., Balan, S., Bonjer, K.-P., Diehl, T., Forbriger, T., Mărmureanu, G., Wenzel, F. & Wirth, W., 2005. Broadband urban seismology in the bucharest metropolitan area, *Seism. Res. Lett.*, **76**(5), 574–580.

Wielandt, E., 2002. Seismic sensors and their calibration. In *New Manual of Seismological Observatory Practice*, eds. P. Bormann & E. Bergmann. GeoForschungsZentrum, Potsdam, Germany. <http://www.geophys.uni-stuttgart.de/seismometry/man_html/index.html>, <http://www.geophys.uni-stuttgart.de/downloads/Postscript-files>.

Ziehm, J. & Forbriger, T., 2006. Seismisches Monitoring einer Großstadt, Analyse von H/V Spektralverhältnissen. In *32. Sitzung der Arbeitsgruppe Seismologie*. Haidhof, Germany.

Zürn, W., 2003. Boussinesque-problem: Point load on an elastic halfspace. A compilation of 12 published solutions to Boussinesque's problem. Unpublished manuscript.

4 Point load on an elastic halfspace

The solution to Boussinesque's problem is (Zürn 2003)

$$u_z(r,z) = -\frac{F}{4\pi\mu R}\frac{1}{R}\left(\frac{\lambda+2\mu}{\lambda+\mu} + \frac{z^2}{R^2}\right) \tag{6}$$

for the vertical component and

$$u_r(r,z) = -\frac{F}{4\pi(\lambda+\mu)}\frac{1}{r}\left(1 + \frac{z}{R} + \frac{\lambda+\mu r^2 z}{\mu \quad R^3}\right) \tag{7}$$

for the horizontal component in cylindrical coordinates with the radial coordinate $r = \sqrt{x^2+y^2}$, the spacial distance $R = \sqrt{r^2+z^2}$ to the load, the force

$$\vec{f} = -F\delta(\vec{x})\hat{e}_z \tag{8}$$

at the origin and λ and μ being Lamé's parameters for the homogeneous halfspace at $z < 0$.
For observations at the surface of the elastic halfspace I simply obtain

$$u_z(r,z=0) = -\frac{F}{4\pi\mu r}\frac{1}{}\frac{\lambda+2\mu}{\lambda+\mu} \tag{9}$$

for the vertical component and

$$u_r(r,z=0) = -\frac{F}{4\pi(\lambda+\mu)}\frac{1}{r} \tag{10}$$

for the horizontal component, which is valid for $r > 0$.

For $F(t)$ varying with time eqs. (9) and (10) are the near-field solution. In this case the seismometer senses

$$a_z(t,r,z=0) = -\frac{\ddot{F}(t)}{r}\frac{\lambda+2\mu}{4\pi\mu(\lambda+\mu)}, \tag{11}$$

$$a_h(t,r,z=0) = -\frac{\ddot{F}(t)}{r}\frac{1}{4\pi(\lambda+\mu)}, \tag{12}$$

and

$$a_\tau(t,r,z=0) = g\frac{\partial u_z(t,r,z)}{\partial r} \tag{13}$$

$$= g\frac{F(t)}{r^2}\frac{\lambda+2\mu}{4\pi\mu(\lambda+\mu)}. \tag{14}$$

Using eq. (5) I obtain

$$A_{\mathrm{H/V}}(\omega) = \frac{\mu}{\lambda+2\mu} + \frac{g}{r\omega^2} \tag{15}$$

for the effect of the point load in the H/V analysis. While it is constant and less than 1 with

$$\lim_{\omega\to\infty} A_{\mathrm{H/V}}(\omega) = \frac{\mu}{\lambda+2\mu} \tag{16}$$

at high frequencies, its contribution

$$\lim_{\omega\to 0} A_{\mathrm{H/V}}(\omega) = \frac{g}{r\omega^2} \tag{17}$$

is increasing with decreasing frequency.
Consider a point load at $r = 10\,\mathrm{m}$ distance, with $\lambda = \mu$ in the halfspace and $g = 9.81\,\mathrm{m\,s^{-2}}$. Then the tilt contribution is larger than 2 at frequencies less than

$$\omega_{\mathrm{limit}} = 2\pi \cdot 0.11\frac{1}{\mathrm{s}} \tag{18}$$

and is likely to mask H/V peaks produced by surface wave ellipticity (Fig. 2).

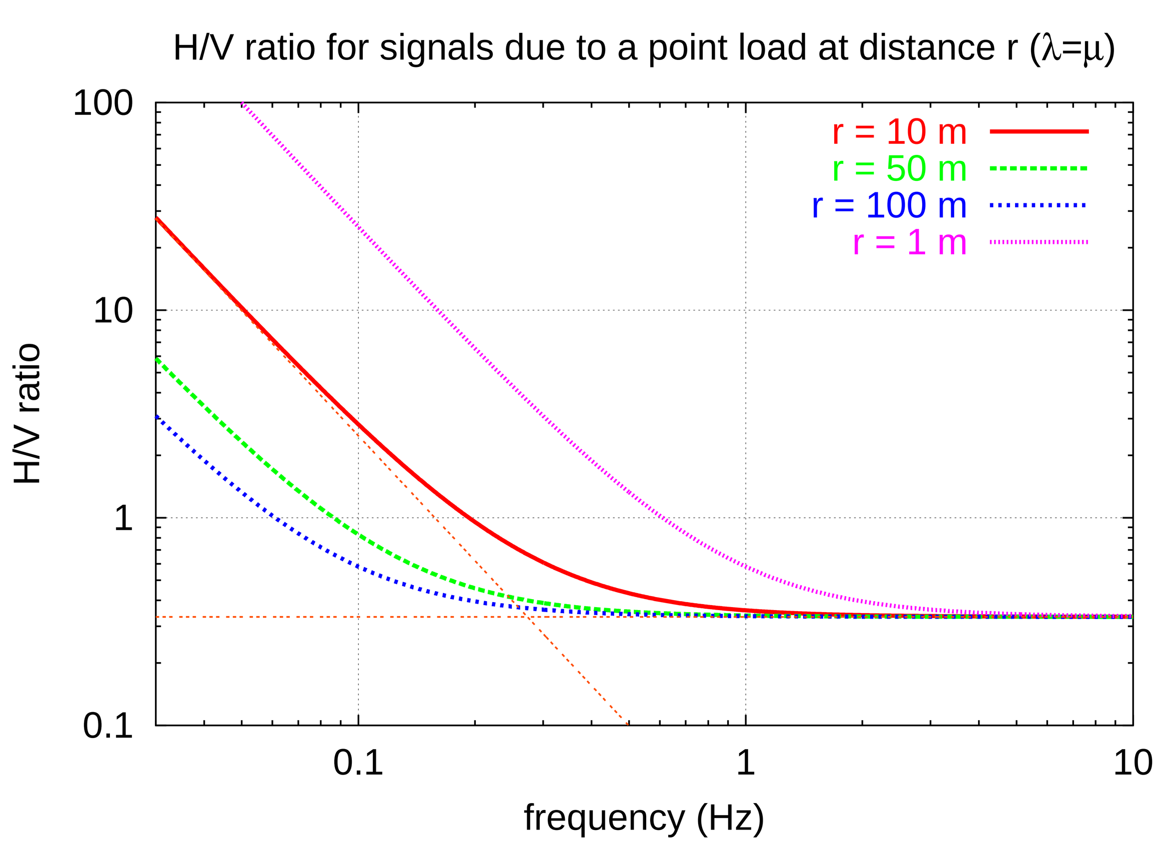

Figure 2: H/V ratio predicted by eq. (15) for a point load at distance r. Lamé's parameters are assumed to be equal ($\lambda = \mu$).

5 Propagating surface wave

The surface displacement of a plane surface wave is

$$u_z(r,t) = A(\omega)\sin(k(\omega)r - \omega t) \tag{19}$$

and

$$u_r(r,t) = A(\omega)\,\varepsilon(\omega)\cos(k(\omega)r - \omega t), \tag{20}$$

with the ellipticity $\varepsilon(\omega)$, where $\varepsilon < 0$ for prograde waves and $\varepsilon > 0$ for retrograde waves. The seismometer senses

$$a_z(r,t) = -\omega^2 A(\omega)\sin(k(\omega)r - \omega t), \tag{21}$$

$$a_r(r,t) = -\omega^2 A(\omega)\,\varepsilon(\omega)\cos(k(\omega)r - \omega t), \tag{22}$$

and

$$a_\tau(r,t) = gk(\omega)A(\omega)\cos(k(\omega)r - \omega t) \tag{23}$$

in this case. Considering the tilt contribution from eq. (23) I obtain

$$A_{\mathrm{H/V}}(\omega) = \left|\frac{g}{\omega c(\omega)} - \varepsilon(\omega)\right|, \tag{24}$$

in the H/V analysis, where $c(\omega) = \omega/k(\omega)$ is the phase velocity.

If $\varepsilon(\omega)$ and $c(\omega)$ are bounded, the H/V ratio

$$\lim_{\omega\to\infty} A_{\mathrm{H/V}}(\omega) = \varepsilon(\omega) \tag{25}$$

provides the ellipticity at high frequencies but is likely to increase with decreasing frequency at the low-frequency limit

$$\lim_{\omega\to 0} A_{\mathrm{H/V}}(\omega) = \frac{g}{\omega c(\omega)}. \tag{26}$$

Inertial and tilt contribution cancel at

$$\omega_C = \frac{g}{\varepsilon c} \tag{27}$$

for retrograde waves.
For a surface wave with $c = 2000\,\mathrm{m\,s^{-1}}$, $\varepsilon = 0.67$, and $g = 9.81\,\mathrm{m\,s^{-2}}$

$$\omega_C = 2\pi \cdot 1.2 \cdot 10^{-3}\frac{1}{\mathrm{s}}. \tag{28}$$

The frequencies for which $A_{\mathrm{H/V}} > 1$ due to the tilt contribution, are not significant for our H/V studies (Fig. 3).

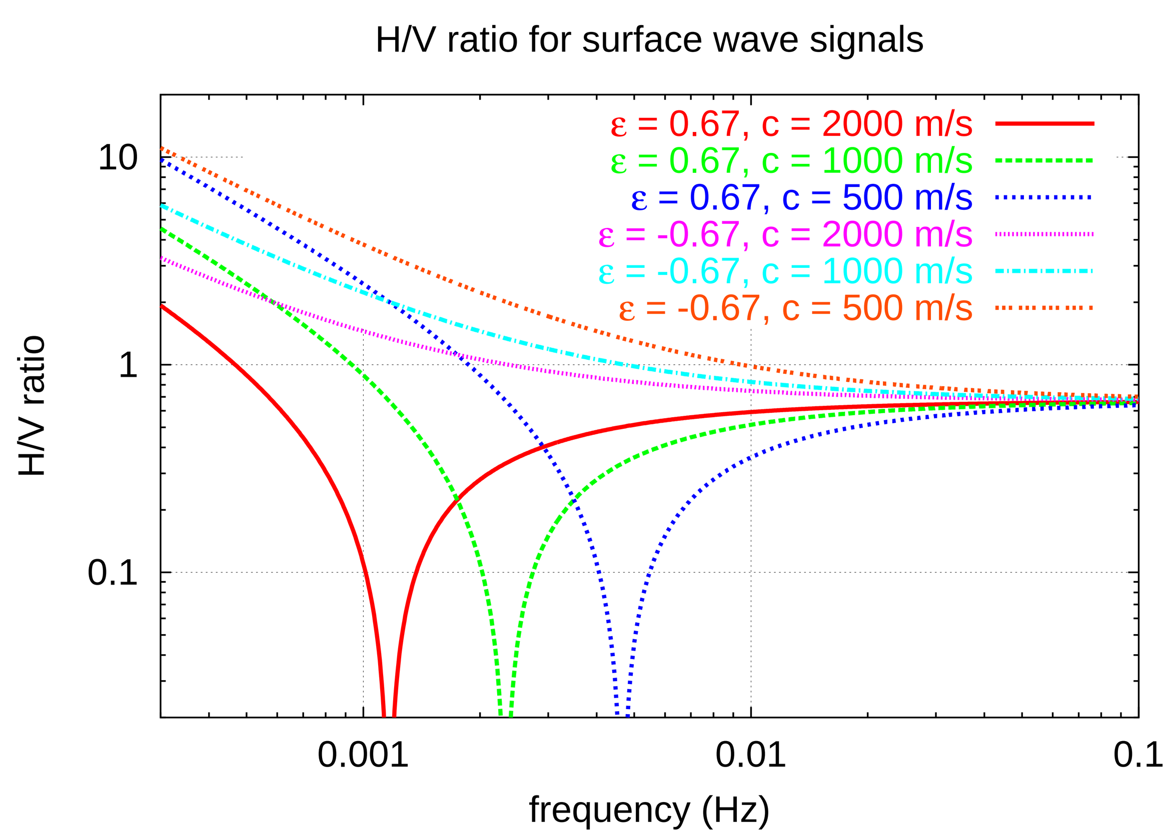

Figure 3: H/V ratio predicted by eq. (24) for surface waves of constant phase velocity c and constant ellipticity ε.

6 Conclusions

In urban environments point loads that vary with time (moving vehicles, buildings loaded by wind, etc.) must be expected to contribute to the long-period seismic signal. While the tilt contribution from surface waves is not significant in the frequency band studied with URS data, tilt due to point loads has the potential to mask surface wave ellipticity at frequencies less than 0.2 Hz. Whether this effect becomes apparent in H/V ratio depends on the relative strength of the point load compared to the amplitude of the microseisms under investigation. This may explain the strong scatter of H/V from different time windows at frequencies below 0.1 Hz in Fig. 1.

Acknowledgements

I'm grateful to Julia Ziehm for providing Fig. 1 and to Walter Zürn and Olivier Sèbe for inspiring discussions. URS data was recorded with the KABBA array within the framework of CRC 461.